Period 4, March 11, 2025

Parallel Line = No Solution
2 Lines cross, one Solution
Same Line, ∞ Solutions

Consider the system:

$$\begin{cases} x + 2y = 2 \\ x - 2y = 6. \end{cases}$$

Determine if each ordered pair is a solution of the system:

a. (4, −1) **b.** (−4, 3).

✓

$4+2(-1)=2$

$4-2=2$

$4-2(-1)=6$

$4+2=6$

True

$-4+2(3)=2$

$-4+6=2$ True

$-4-2(3)=6$

$-4-6\neq 6$ False

$-10\neq 6$

No Sol

Solving Linear Systems by Substitution

1. Solve either of the equations for one variable in terms of the other. (If one of the equations is already in this form, you can skip this step.)
2. Substitute the expression found in step 1 into the *other* equation. This will result in an equation in one variable.
3. Solve the equation containing one variable.
4. Back-substitute the value found in step 3 into one of the original equations. Simplify and find the value of the remaining variable.
5. Check the proposed solution in both of the system's given equations.

Solve by the substitution method:

$$\begin{cases} 5x - 4y = 9 \\ x - 2y = -3. \end{cases}$$

$x-2y=-3$

$+2y \quad +2y$

$x=2y-3$

$5=2(4)-3=8-3$

$5(2y-3)-4y=9$

$10y-15-4y=9$

$+15 \qquad +15$

$\frac{\cancel{6}y}{\cancel{6}}=\frac{24}{6} \Rightarrow y=4$

(5, 4)

Test

$5(5)-4(4)$

$25-16=9$

$5-2(4)$

$5-8=-3$

work

CHECK POINT 2 Solve by the substitution method:

$$\begin{cases} 3x + 2y = 4 \\ 2x + y = 1. \end{cases} \Rightarrow \boxed{y = 1 - 2x} = 1 - 2(-2) = 1 + 4 = 5$$

$3x + 2(1 - 2x) = 4$

$3x + \cancel{2} - 4x = 4$
$\quad -\cancel{2} \qquad\quad -2$

$-x = 2$

$x = -2$

Solution
$(-2, 5)$

Solving Linear Systems by Addition

1. If necessary, rewrite both equations in the form $Ax + By = C$.
2. If necessary, multiply either equation or both equations by appropriate nonzero numbers so that the sum of the x-coefficients or the sum of the y-coefficients is 0.
3. Add the equations in step 2. The sum is an equation in one variable.
4. Solve the equation in one variable.
5. Back-substitute the value obtained in step 4 into either of the given equations and solve for the other variable.
6. Check the solution in both of the original equations.

Use elimination

$$\begin{cases} 3x - 4y = 11 \\ -3x + 2y = -7. \end{cases}$$

$0x + -2y = 4$

$-2y = 4$

$y = -2$

$3x - 4(-2) = 11$

$3x + 8 = 11$
$\quad -8 \quad -8$

$3x = 3$

$\boxed{x = 1}$

Test

$3(1) - 4(-2) = 3 + 8 = 11$

$-3(1) + 2(-2) = -3 - 4 = -7$

work!

Solve by the addition method:

$$\begin{cases} (3x + 2y = 48) \cdot 4 \\ 9x - 8y = -24. \end{cases}$$

-get Rid of y's

$$12x+8y=192$$
$$9x-8y=-24$$
$$21x+0y=168$$
$$\frac{21x}{21}=\frac{168}{21}$$
$$x=8$$

$$3x+2y=48$$
$$3(8)+2y=48$$
$$24+2y=48$$
$$\cancel{-24} \quad -24$$
$$2y=24$$
$$y=12$$

CHECK POINT 3 Solve by the addition method:

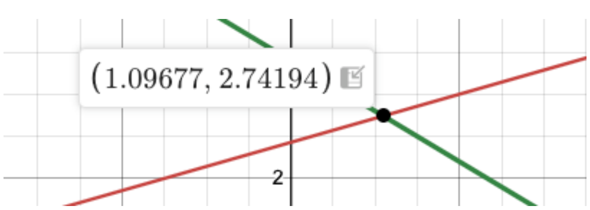
$$\begin{cases} 4x + 5y = 3 \\ (2x - 3y = 7)(-2) \end{cases}$$

$$4x+5y=3$$
$$-4x+6y=-14$$
$$0x+11y=-11$$
$$y=-1$$

$$4x+5(-1)=3$$
$$4x-\cancel{5}=3$$
$$+\cancel{5} \quad +5$$
$$4x=8$$
$$x=2$$

Solve by the addition method:

$$\begin{cases} 2x = 7y - 17 \Rightarrow (2x-7y=-17)3 \\ 5y = 17 - 3x. \Rightarrow (3x+5y=17)(-2) \end{cases}$$



$$6x-21y=-51$$
$$-6x-10y=-34$$
$$0x-31y=-85$$
$$y=\frac{85}{31}=2.7419$$

CHECK POINT 4 Solve by the addition method:

$$\begin{cases} 2x = 9 + 3y \\ 4y = 8 - 3x. \end{cases}$$

$\Rightarrow (2x-3y=9)4 \Rightarrow 8x-12y=36$

$(3x+4y=8)3 \quad 9x+12y=24$

$17x = 60$

$x = \frac{60}{17}$

The Number of Solutions to a System of Two Linear Equations

The number of solutions to a system of two linear equations in two variables is given by one of the following. (See **Figure 7.3**.)

Number of Solutions	What This Means Graphically
Exactly one ordered-pair solution	The two lines intersect at one point.
No solution	The two lines are parallel.
Infinitely many solutions	The two lines are identical.

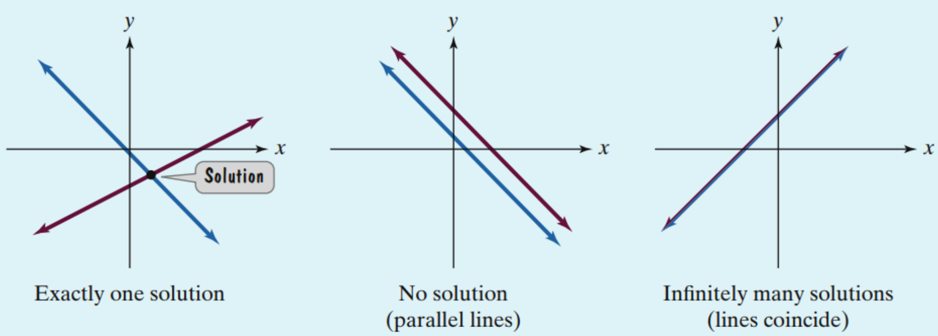
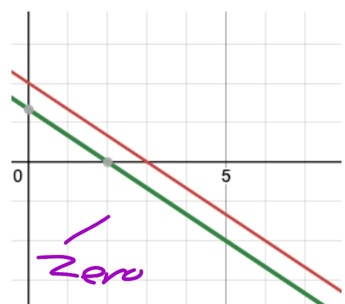


Figure 7.3 Possible graphs for a system of two linear equations in two variables

Solve the system:

$$\begin{cases} 4x + 6y = 12 \\ 6x + 9y = 12. \end{cases}$$



Zero solutions

CHECK POINT 5 Solve the system:

$$\begin{cases} 5x - 2y = 4 \\ -10x + 4y = 7. \end{cases}$$

Solve the system:

$$\begin{cases} y = 3x - 2 \\ 15x - 5y = 10. \end{cases}$$

Strategy for Problem Solving Using Systems of Equations

Step 1 Read the problem carefully. Attempt to state the problem in your own words and state what the problem is looking for. Use variables to represent unknown quantities.

Step 2 Write a system of equations that models the problem's conditions.

Step 3 Solve the system and answer the problem's question.

Step 4 Check the proposed solution in the original wording of the problem.

Revenue and Cost Functions

A company produces and sells x units of a product.

Revenue Function

$$R(x) = (\text{price per unit sold})x$$

Cost Function

$$C(x) = \text{fixed cost} + (\text{cost per unit produced})x$$

The Profit Function

The profit, $P(x)$, generated after producing and selling x units of a product is given by the **profit function**

$$P(x) = R(x) - C(x),$$

where R and C are the revenue and cost functions, respectively.

✓ CHECK POINT 7 A company that manufactures running shoes has a fixed cost of \$300,000. Additionally, it costs \$30 to produce each pair of shoes. They are sold at \$80 per pair.

a. Write the cost function, C, of producing x pairs of running shoes.

b. Write the revenue function, R, from the sale of x pairs of running shoes.

c. Determine the break-even point. Describe what this means.

$C(x) = 300{,}000 + 30x$

$R(x) = 80 \cdot x$

$\text{Profit} = 80x - (300{,}000 + 30x)$

$80x - 30x - 300{,}000$

Break Even

$0 = 50x - 300{,}000$

$\frac{300{,}000}{50} = \frac{\cancel{50}x}{\cancel{50}}$

$6{,}000 = x$

A chemist working on a flu vaccine needs to mix a 10% sodium-iodine solution with a 60% sodium-iodine solution to obtain 50 milliliters of a 30% sodium-iodine solution. How many milliliters of the 10% solution and of the 60% solution should be mixed?

Solution

Step 1 Use variables to represent unknown quantities.

Let x = the number of milliliters of the 10% solution to be used in the mixture.

Let y = the number of milliliters of the 60% solution to be used in the mixture.

$x + y = 50 \Rightarrow x = y - 50$

$0.1x + 0.6y = .30(50)$

$0.1(y-50) + 0.6y = 15$

$0.7y - \cancel{5} = 15$

$+\cancel{5} \quad +5$

$\frac{\cancel{0.7}y}{\cancel{0.7}} = \frac{20}{0.7}$

$y = 28.57$

$x = 50 - 28.57 = 21.43$

When a small airplane flies with the wind, it can travel 450 miles in 3 hours. When the same airplane flies in the opposite direction against the wind, it takes 5 hours to fly the same distance. Find the average velocity of the plane in still air and the average velocity of the wind.

A plane is flying at a speed of 330 miles per hour on a bearing of N75°E. Its ground speed is 390 miles per hour and its true course, given by the direction angle of the ground speed vector, is 30°. Find the speed, in miles per hour, and the direction angle, in degrees, of the wind.

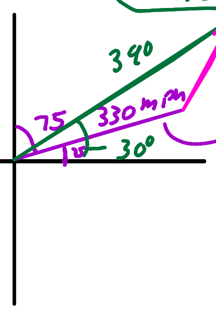
The speed of the wind is ☐ miles per hour.
(Do not round until the final answer. Then round to the nearest tenth as needed.)

$x = 390 \cdot \cos 30 = 337.75$

$y = 390 \cdot \sin 30 = 195$

$337.75\hat{i} + 195\hat{j}$

wind $a\hat{i} + b\hat{j}$



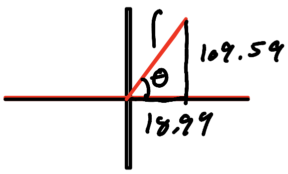
$x = 330 \cdot \cos 15 = 318.76$

$y = 330 \cdot \sin 15 = 85.41$

$318.76\hat{i} + 85.41\hat{j}$

$r^2 = 109.54^2 + 18.99^2$

111.2



$318.76\hat{i} + a\hat{i} = 337.75\hat{i}$ $\quad a = 18.99\hat{i}$

$85.41\hat{j} + b\hat{j} = 195\hat{j}$ $\quad b = 109.54$

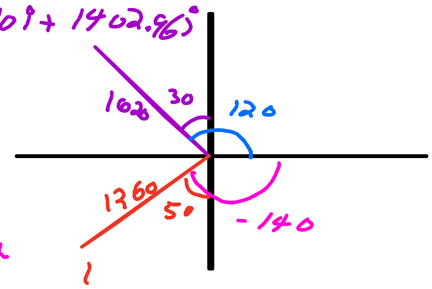
The magnitude and direction exerted by two tugboats towing a ship are 1620 pounds, N30°W, and 1260 pounds, S50°W, respectively. Find the magnitude, in pounds, and the direction angle, in degrees, of the resultant force.

add

The magnitude of the resultant force is 1871.66 pounds.
(Do not round until the final answer. Then round to the nearest hundredth as needed.)

The direction angle of the resultant force is 161.5°.
(Do not round until the final answer. Then round to the nearest tenth as needed.)

$-810\hat{i} + 1402.96\hat{j}$



$1620 \cdot \cos 120 = -810$

$1620 \cdot \sin 120 = 1402.96$

$1260 \cdot \cos(-140) = -965.22$

$1260 \cdot \sin(-140) = -809.91$

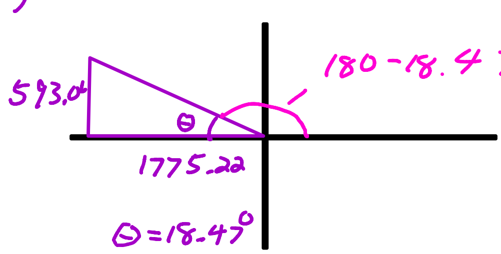
$-965.22\hat{i} + -809.91\hat{j}$

Result

$-810\hat{i} + 1402.96\hat{j} - 965.22\hat{i} - 809.91\hat{j} = -1775.22\hat{i} + 593.06\hat{j}$

$\sqrt{(593.06^2 + 1775.22^2)} = 1871.66$

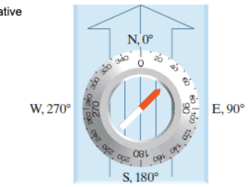
$\tan^{-1} = \frac{593.06}{1775.22}$



$180 - 18.47 = 161.5$

$\theta = 18.47°$

An airplane has an air speed of 275 miles per hour and a compass heading of 140°. A steady wind of 56 miles per hour is blowing in the direction of 197°. What is the plane's true speed relative to the ground? What is its compass heading relative to the ground?



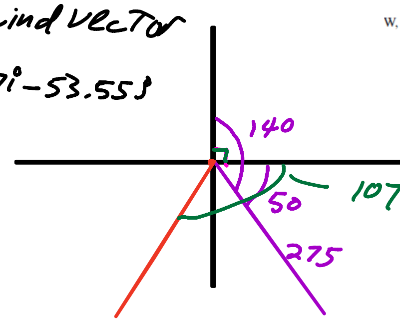
groundspeed = Airplane vector + wind vector

$176.77i - 210.66j + -16.37i - 53.55j$

$160.4i - 264.21j$

$\sqrt{160.4^2 + 264.21^2}$

309.09



The groundspeed is $\boxed{}$ miles per hour.

(Round to the nearest integer as needed.)

$56 \cdot \cos(-107) = -16.37$

$56 \cdot \sin(-107) = -53.55$

$275 \cdot \cos(-50) = 176.77$

$275 \cdot \sin(-50) = -210.66$

wind $= -16.37i - 53.55j$

Airplane $= 176.77i - 210.66j$

Let $\mathbf{u} = 2\mathbf{i} - 2\mathbf{j}$, and $\mathbf{w} = -\mathbf{i} - 8\mathbf{j}$. Find $\|\mathbf{w} - \mathbf{u}\|$.

$w - u$

$-i - 8j - (2i - 2j)$

$-i - 8j - 2i + 2j = -3i - 6j$

$\|\mathbf{w} - \mathbf{u}\| = 3\sqrt{5}$ (Type an exact answer, using radicals as needed.)

$\|w - u\| = \|-3i - 6j\|$

$\sqrt{3^2 + 6^2}$

$\sqrt{9 + 36} = \sqrt{45}$

$\sqrt{3 \cdot 3 \cdot 5}$

$3\sqrt{5}$

A vector is described. Express the vector in terms of **i** and **j**.

A plane flying downward toward a runway at 148 miles per hour makes an angle of 4° with the runway.

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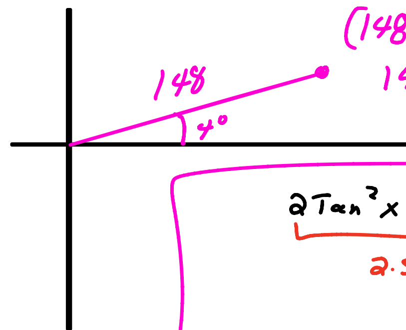
v = 147.6i + 10.3j

(Simplify your answer. Round to the nearest tenth as needed. Type your answer in terms of i and j.)



$(148\cos 4^\circ)\mathbf{i} + 148\sin(4)\mathbf{j}$

$147.64\mathbf{i} + 10.32\mathbf{j}$

$$2\tan^2 x - 7\tan x + 5 = 2\tan^2 x - 2\tan x - 5\tan x + 5$$

$2 \cdot 5 = 10$

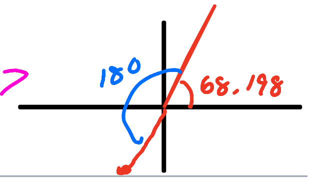
$-2 \quad -5$

$2\tan x(\tan x - 1)$

$-5(\tan x - 1)$

$\frac{\pi}{4} = .7853975$

$\frac{5\pi}{4} = 3.926990817$



Use a calculator to solve the equation on the interval $[0, 2\pi)$.

$2\tan^2 x - 7\tan x + 5 = 0$

$(\tan x - 1)(2\tan x - 5) = 0$

Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

$\tan x = 1$

$x = \frac{\pi}{4}, \frac{5\pi}{4}$

$\tan x = \frac{5}{2}$

$\tan^{-1}\frac{5}{2} = 68.198^\circ$

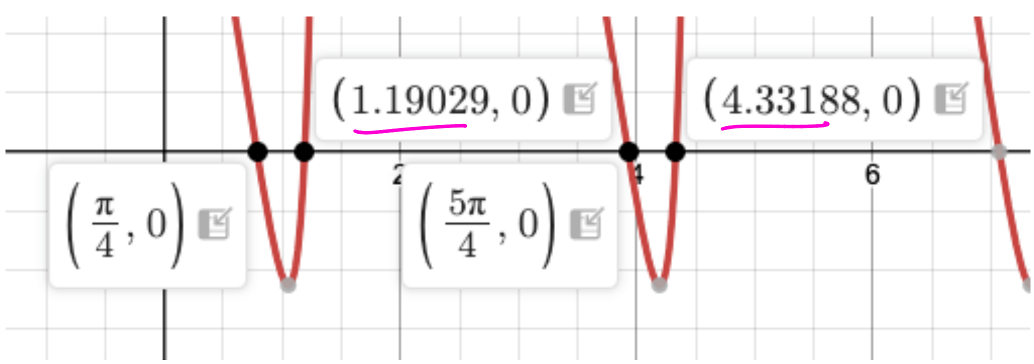
A. x = 0.7854,1.1903,3.9270,4.3319

(Type your answer in radians. Type an integer or decimal rounded to four decimal places as needed. Use a comma to separate answers as needed.)

B. There is no solution.

or 1.190 R

$+\pi$



The forces $\mathbf{F}_1, \mathbf{F}_2, \mathbf{F}_3, \ldots, \mathbf{F}_n$ acting on an object are in equilibrium if the resultant force is the zero vector.

$$\mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 + \cdots + \mathbf{F}_n = \mathbf{0}$$

The forces $\mathbf{F}_1 = 4\mathbf{i} - 4\mathbf{j}$ and $\mathbf{F}_2 = 8\mathbf{i} + 2\mathbf{j}$ are acting on an object.

a. Find the resultant force. $4i - 4j + 8i + 2j = 12i - 2j$

b. What additional force is required for the given forces to be in equilibrium?

$-12i + 2j$ $\quad = 0$

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a. The resultant force is $12i - 2j$.

(Simplify your answer. Type your answer in terms of i and j.)

b. The additional force required for the given forces to be in equilibrium is $-12i + 2j$.

(Simplify your answer. Type your answer in terms of i and j.)

Find the unit vector that has the same direction as the vector **v**.

$\mathbf{v} = \mathbf{i} + 2\mathbf{j}$ $\qquad \|v\| = \sqrt{1^2 + 2^2} = \sqrt{5}$

The unit vector that has the same direction as the vector $\mathbf{v} = \mathbf{i} + 2\mathbf{j}$ is $\dfrac{\sqrt{5}}{5}i + \dfrac{2\sqrt{5}}{5}j$.

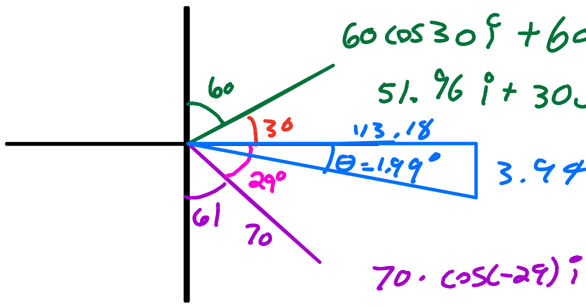
$$\text{Unit vector} = \frac{i + 2j}{\sqrt{5}} = \frac{i\sqrt{5}}{\sqrt{5}\cdot\sqrt{5}} + \frac{2j\cdot\sqrt{5}}{\sqrt{5}\cdot\sqrt{5}}$$

$$\frac{i\sqrt{5}}{5} + \frac{2j\sqrt{5}}{5}$$

The magnitude and direction of two forces acting on an object are 70 pounds, S61°E, and 80 pounds, N60°E, respectively. Find the magnitude, to the nearest hundredth of a pound, and the direction angle, to the nearest tenth of a degree, of the resultant force.

The magnitude is approximately ☐ pounds.

(Do not round until the final answer. Then round to the nearest hundredth as needed.)



$60\cos 30\,\hat{i} + 60\sin 30\,\hat{j}$

$51.96\,\hat{i} + 30\,\hat{j}$

$S(90-1.99)E$

$S\ 88.01°E$

Mag 113.25

$70\cdot\cos(-29)\,\hat{i} + 70\sin(-29)\,\hat{j}$

$+61.22\,\hat{i} - 33.94\,\hat{j}$

$51.96\,\hat{i} + 61.22\,\hat{i} + 30\,\hat{j} - 33.94\,\hat{j}$

$113.18\,\hat{i} - 3.94\,\hat{j}$

$\sqrt{(113.18)^2 + (-3.94)^2} = 113.25$

Write the vector **v** in terms of **i** and **j** whose magnitude and direction angle are given.

$\|\mathbf{v}\| = \frac{6}{7},\ \theta = 118°$

$-0.4024\,\hat{i} + 0.7568\,\hat{j}$

$\frac{6}{7}\cdot\cos 118 = -0.4024$

$\frac{6}{7}\sin 118 = 0.7568$

Let **u** = −4**i** + 9**j**, **v** = 5**i** − **j**, and **w** = −4**i**. Find 5**u** − (5**v** − **w**).

$5(-4\hat{i} + 9\hat{j}) - (5(5\hat{i} - \hat{j}) - (-4\hat{i}))$

$-20\hat{i} + 45\hat{j} - (25\hat{i} - 5\hat{j} + 4\hat{i})$

$-20\hat{i} + 45\hat{j} - (29\hat{i} - 5\hat{j})$

$-20\hat{i} + 45\hat{j} - 29\hat{i} + 5\hat{j} = -49\hat{i} + 50\hat{j}$

Find the magnitude $\|\mathbf{v}\|$ and the direction angle θ for the given vector $\mathbf{v}$.

$\mathbf{v} = -4\mathbf{i} + 11\mathbf{j}$

$\sqrt{(-4)^2+11^2} = \sqrt{16+121} = \sqrt{137} = 11.70$

$\|\mathbf{v}\| =$ 11.70

(Round to the nearest hundredth as needed.)

$\tan^{-1}\frac{11}{4} = 70.017$

$\theta =$ 110 °

(Round to the nearest tenth as needed.)

$180 - 70.02 = 110$